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Zeeman Effect

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Abstract

The goal of this experiment is to observe and understand the transverse and longitudinal Zeeman effect. Therefore, the splitting of the two σ-lines in the transverse Zeeman effect will be measured. Further, Bohr's magneton is determined and the light emitted along the direction of the magnetic field, the longitudinal Zeeman effect, will be qualitatively analyzed.

Contents

Introduction

The Zeeman effect describes the splitting up of the central spectral lines of atoms when a magnetic field is applied. The normal Zeeman effect is the simplest, it describes the splitting up of one spectral line into three components. A cadmium spectral lamp is used. The splitting up of the red cadmium line (643.8nm) is investigated with a Fabry-Perot interferometer by applying different magnetic flux densities. The Bohr's magneton can be yielded from the evaluation of the results.

1 Theory

In 1862 Michael Faraday unsuccessfully investigated if a magnetic field changes the spectrum of colored flames. Only in 1885 Belgian Fievez was able to show this effect. This was forgotten until it was rediscovered by Pieter Zeeman in 1896. He worked together with Hendrik Lorentz.

The measurements of this experiment were important for the development of atomic shell theory. The splitting of the Cd-spectral line λ=643.8nm into three lines is called Lorentz triplets. This occurs due to the fact that the Cd-atom represents a singlet system with spin S=0. In absence of a magnetic field there exists only one transition between D and P (see Figure 1), the 643.8nm transition.

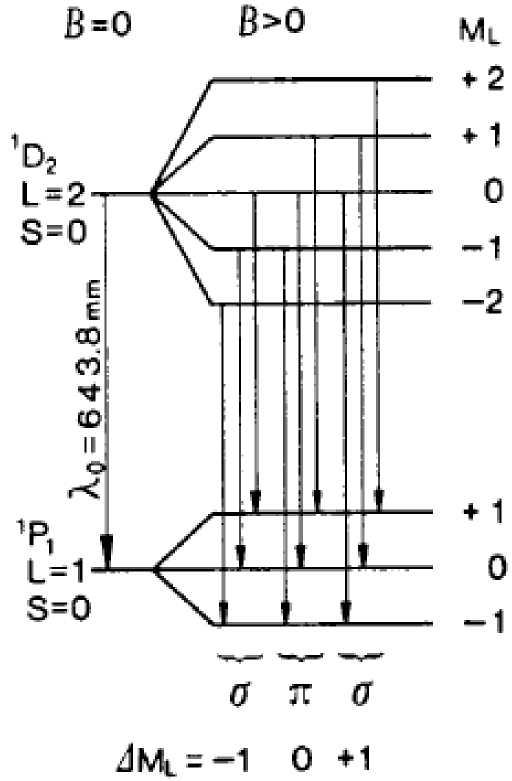

Figure 1: Splitting up of the central spectral lines into the components due to magnetic field.

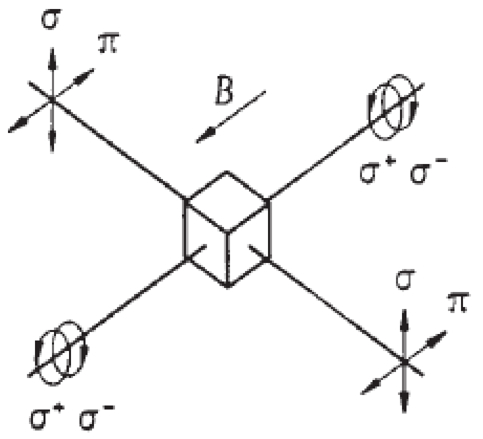

Figure 2: Longitudinal and transverse Zeeman effect.

The magnetic field causes the energy level to split up into 2L+1 components. Radiation transitions between the components are possible if the selection rules

$$\Delta M_L = +1 \qquad \Delta M_L = 0 \qquad \Delta M_L = -1$$

are fulfilled. There are nine permitted transitions in total which can be assigned to three different energies, three transitions for each energy. Thus, only three lines will be visible.

The first three transitions with $\Delta M_L = -1$ give a σ-line whose light is vertically polarized to the magnetic field (see Figure 2). The light of the second group with $\Delta M_L = 0$ is parallel and the light of the $\Delta M_L = +1$ is again vertically polarized to the magnetic field.

Each ring which could be observed without a magnetic field splits up into three rings when a magnetic field is applied perpendicular to the light. Due to the different polarization of the rings an analyzer can be used to filter out the π-lines or the σ-lines depending on the analyzer's polarization. If the analyzer is in vertical position only the σ-lines are visible and if it is horizontal orientated the π-lines can be observed exclusively. This is the transverse Zeeman effect. The polarization of the light propagating parallel to the magnetic field is circular. This means independent of the position of the analyzer two rings appear.

In the case of the transverse Zeeman effect the splitting of the two σ-lines increases with the magnetic field strength. The splitting can be analyzed by an Fabry-Perot interferometer. It is characterized in terms of numbers of wavelengths with a precision of 0.002nm. The Fabry-Perot ètalon consists of two parallel flat glass plates separated by a distance t (see Figure 3). The surfaces pointing inside are coated with a partially transmitting metallic layer.

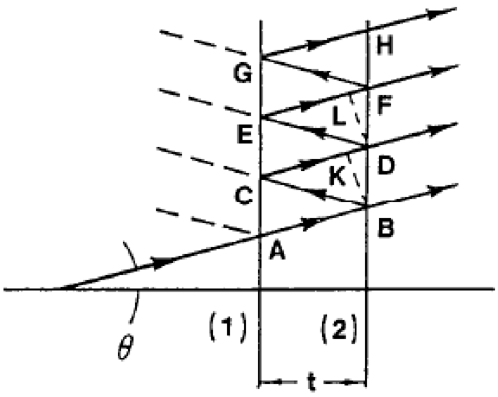

Figure 3: Transmitted and reflected rays at the étalon's parallel surfaces (1) and (2).

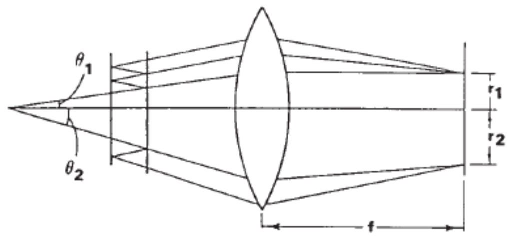

Figure 4: Focusing of the light coming from the Fabry-Pérot étalon. The light is focused onto a ring with the radius $r = f\Theta$, with Θ being the incident angle of the light into the étalon and f the focal length of the lens.

An incident ray under the angle Θ with respect to the normal to the glass plates is split into the paths AB, CD, EF, etc. The path difference between for example AB and CD is

$$\delta = BC + CK$$

where BK is normal to CD.

$$CK = BC\cos 2\Theta \quad \text{and} \quad BC\cos\Theta = t$$

This leads to

$$\delta = BCK = BC(1+\cos 2\Theta) = 2BC\cos^2\Theta = 2t\cos\Theta$$

and for constructive interference

$$n\lambda = 2t \cos \Theta$$

must be fulfilled with n an integer. If the refractive index of the medium between the glass plates is $\mu \neq 1$ the formula needs to be modified:

$$n\lambda = 2\mu t \cos \Theta. \tag{1}$$

Equation 1 is called the basic interferometer equation. To focus the parallel rays B,D,F, etc. they are focused with the help of a lens with the focal length f (see Figure 4).

If Θ obeys Equation 1 the radius of the rings appearing in the focal plane are given by

$$r_n = f \tan \Theta_n \simeq f\Theta_n. \tag{2}$$

Equation 2 is only valid for small angle, that means for rays nearly parallel to the optical axis.

Since

$$n = \frac{2\mu t}{\lambda} \cos \Theta_n = n_0 \cos \Theta_0 = n_0 \left(1 - 2\sin^2 \frac{\Theta_n}{2}\right)$$

with

$$n_0 = \frac{2\mu t}{\lambda}$$

the final formula is

$$n = n_0 \left(1 - \frac{\Theta_n^2}{2}\right) \qquad \text{or} \qquad \Theta_n = \sqrt{\frac{2(n_0 - n)}{n_0}} \tag{3}$$

The center interference n_0 is in general not an integer. n_1 describes the interference of the first ring, with $n_1 < n_0$ due to $n_1 = n_0 \cos \Theta_1$. n_0 can be expressed through the nearest integer n_1

$$n = n_0 - \epsilon \qquad 0 < \epsilon < 1.$$

Therefore, the p-th ring of the interference pattern has

$$n_p = (n_0 - \epsilon) - (p - 1). \tag{4}$$

Using Equation 2, 3 and 4 one gets the radii of the rings

$$r_p = \sqrt{\frac{2f^2}{n_0}} \cdot \sqrt{(p-1)+\epsilon}. \tag{5}$$

The difference between the squares of the radii of two adjacent rings is constant:

$$r_{p+1}^2 - r_p^2 = \frac{2f^2}{n_0} \tag{6}$$

ϵ can be determined by plotting r_p^2 versus p and extrapolating to $r_p^2 = 0$.

In the case where the spectral line is split into two components with wavelength λ_a and λ_b, the components will have fractional orders at the center ϵ_a and ϵ_b:

$$\epsilon_a = \frac{2\mu t}{\lambda_a} - n_{1,a} = 2\mu t \overline{v}_a - n_{1,a} \qquad \epsilon_b = \frac{2\mu t}{\lambda_b} - n_{1,b} = 2\mu t \overline{v}_b - n_{1,b}$$

$n_{1,a}$ and $n_{1,b}$ are the interference orders of the first ring. If the rings do not completely overlap $n_{1,a} = n_{1,b}$. Thus, the difference between the wave numbers of two components is

$$\Delta\overline{v} = \overline{v}_a - \overline{v}_b = \frac{\epsilon_a - \epsilon_b}{2\mu t} \tag{7}$$

From Equation 5 and 6 one gets

$$\frac{r_{p+1,a}^2}{r_{p+1}^2 - r_p^2} - p = \epsilon \tag{8}$$

For the components a and b this yields

$$\frac{r_{p+1,a}^2}{r_{p+1,a}^2 - r_{p,a}^2} - p = \epsilon_a \qquad \frac{r_{p+1,b}^2}{r_{p+1,b}^2 - r_{p,b}^2} - p = \epsilon_b$$

By substituting the last result into Equation 7 one gets the difference of the wave numbers:

$$\Delta\overline{v} = \frac{1}{2\mu t}\left(\frac{r_{p+1,a}^2}{r_{p+1,a}^2 - r_{p,a}^2} - \frac{r_{p+1,b}^2}{r_{p+1,b}^2 - r_{p,b}^2}\right) \tag{9}$$

From Equation 6 one knows that the difference of the radii component a squared is equal to one of component b.

$$\Delta_a^{p+1,p} = r_{p+1,a}^2 - r_{p,a}^2 = \frac{2f^2}{n_{0,a}} \qquad \Delta_b^{p+1,p} = r_{p+1,b}^2 - r_{p,b}^2 = \frac{2f^2}{n_{0,b}}$$

Thus, whatever the value of p is,

$$\Delta_a^{p+1,p} = \Delta_b^{p+1,p}$$

and similar all values

$$\delta_{a,b}^{p+1,p} = r_{p+1,a}^2 - r_{p+1,b}^2$$

must be equal. δ and Δ denote the average differences of the waver numbers of the components a and b, setting $\mu = 1$.

This leads to the evidence that $\Delta\overline{v}$ is neither depending on the dimension of the radii not the amplification of the interference pattern.

$$\Delta\overline{v} = \frac{1}{2t}\frac{\delta}{\Delta} \tag{10}$$

2 Setup

Figure 5: Experimental setup.

The electromagnet is placed on a rotating table what enables to switch between transverse and longitudinal Zeeman measurements (see Figure 5). The two pole-shoes with holes in it are mounted in way that a gap of 9mm remains for the Cd-lamp (see Figure 6). The pole-shoes must be tightened such that they do not move during the experiments where a magnetic field is applied. The Cd-lamp is connected to the power supply and put into the the gap without touching the pole-shoes. The coils of the electromagnet are connected in parallel and with an ammeter in between to the variable power supply (up to 20VDC, 12A). To smooth the DC-voltage a 22'000μF capacitor is installed in parallel to the power output.

For the investigation of the line splitting several optical components are necessary (see Figure 6). The diaphragm is only used for the transverse Zeeman effect where it acts as light source. The lens L_1 together with an internal lens (f=100mm) of the ètalon create a nearly parallel light beam which is needed for proper interference pattern. An interchangeable color filter of the ètalon filters the 643.8nm cadmium line out. Lens L_2 focuses the interference pattern of rings on the screen. The ring pattern is observed through lens L_3 and the ring diameters can be measured with the scale. The scale can be moved with a precision of 1/100 of an millimeter.

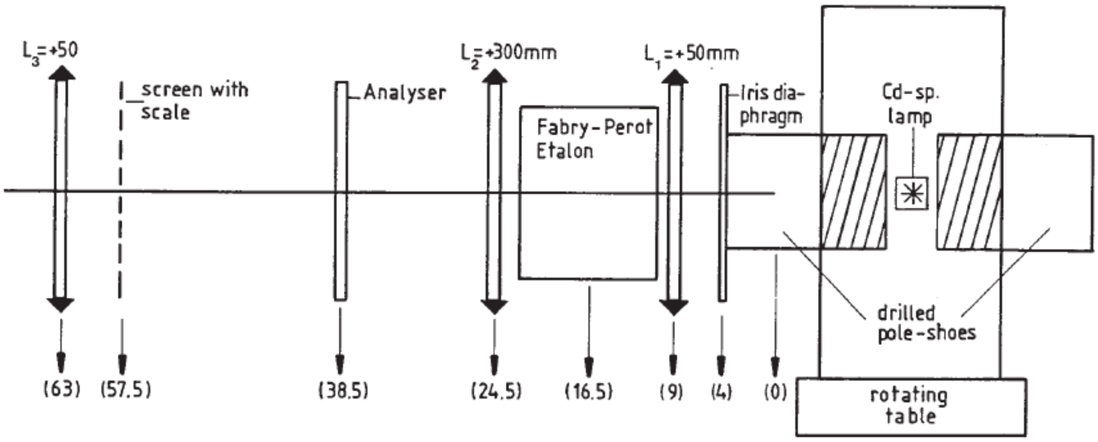

Figure 6: Scheme of the optical components

The rotating table together with the electromagnet, the pole-shoes and the Cd-lamp are installed at a height of 16cm. With the help of the spirit level the electromagnet is brought into a perfectly horizontal position. The optical bench with all elements as in Figure 6 except for the iris diaphragm is placed closer to the electromagnet, such that one of the outlet holes of the pole-shoes is placed where the diaphragm usually is. L_1 is adjusted in a way that the outlet is in its focus. After, all other optical components are readjusted too. The current through the coils is set to 8A for some time and the interference pattern in axial direction is observed through L_3. With a last adjustment of the ètalon (slightly left/right) and displacement of L_2 a sharp and centered pattern is generated. The slash of the scale representing "0" should be in the center of the most inner ring.

Now, the electromagnet is rotated by 90°, the diaphragm is inserted and the analyzer is turned until the π-line disappears and the two σ-lines appear.

3 Measurements and Analysis

First the calibration of the magnetic flux needs to be done. A teslameter is used to determine the magnetic flux at the gap where usually the Cd-lamp is. The measured values for the magnetic flux are plotted against the applied electric current to the coils (see Figure 7).

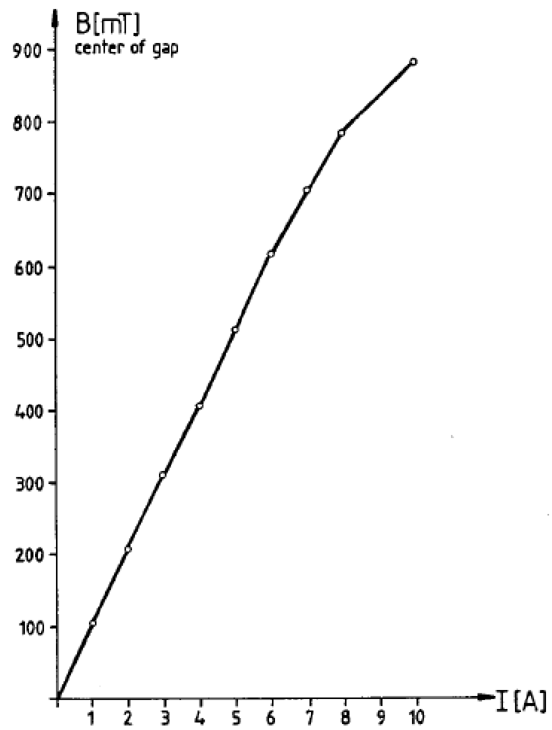

Figure 7: Magnetic flux B in between the two Helmholtz coils, where for the measurements a Cd-lamp is installed, depending on the coil current.

1. When a proper ring pattern is achieved the diameter of a ring can be measured. To do so the scale is shifted horizontally until the "0" slash of the scale for example crosses the fourth ring to the left. A magnetic field corresponding to a 4A coil current is applied and the splitting of the rings observed. The analyzer is orientated vertically what causes only the σ-lines to appear. The "0" slash is brought to the middle of the outer ring. Next, the inner σ-line is measured and so forth until the "0" slash is placed at the outer σ-line of the fourth ring to the right. The last reading minus the first reading divided by two provides the radius $r_{4,b}$. With the same method the radii of the other rings $r_{4,a}$, $r_{3,b}$, $r_{3,a}$, $r_{2,b}$, $r_{2,a}$, $r_{1,b}$ and $r_{1,a}$ should be evaluated. Further measurements should be done with the same method but with different coil currents, for instance 5A, 6A, 8A and 10A.

2. The wave number difference of a σ-line with respect to the central line is $\Delta\overline{v}/2$. The energy of the radiation electrons changes by

$$\Delta E = E_{L,M_L} - E_{L-1,M_L-1} = hc\frac{\Delta\overline{v}}{2}. \tag{11}$$

The change in energy ΔE is proportional to the magnetic flux density B with the Bohr's magneton μ_B as proportionality:

$$\Delta E = \mu_B B \tag{12}$$

Extract the Bohr's magneton from the measured data.
The literature value for the Bohr's magneton is $\mu_{B,Lit.} = (9.273)10^{-24}\frac{\mathrm{J}}{\mathrm{T}}$.

3. For the observation of the longitudinal Zeeman effect the electromagnet is turned by 90°. When a magnetic field is applied (8A coil current) each ring splits up into two, no matter at which position the analyzer is.
To show that the light of longitudinal Zeeman effect is circular polarized a $\lambda/4$ plate is used to transfer into circular polarized light. When the optical axis of the $\lambda/4$ plate is vertically orientated and the analyzer is placed at an angle of +45° with respect to the vertical axis, one ring disappears. If the analyzer is installed with an angle of -45° the other ring disappears. Explain the effect.

Note: All figures are taken from PHYWE LEP 5.1.10.