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Dielectric Constant with Microwaves

Fortgeschrittenenpraktikum I/II

June 20, 2017

Abstract

The goal of this experiment is to determine the dependency of the complex dielectric constant of tert-Butyl chloride on the temperature. To get tert-Butyl chloride into the temperature range of -70 to -50°C, where the substance shows interesting behaviour, a cooling bath consisting of dry ice and acetone is needed. The complex dielectric constant or permitivity can be calculated by measuring the standing wave pattern of microwaves inside a waveguide with a thin layer of tert-Butyl chloride on one end.

Contents

1 Introduction

The static dielectric constant of a material is usually measured with a capacitor using this material as a dielectric. However, when using electromagentic waves, the dielectric properties of materials can change. In fact, the (complex) dielectric constant of any material depends on many parameters, most importantly the frequency and the temperature. When measuring the dielectric properties of a material at high frequencies, another method than a capacitor is needed. In this case we are using a klystron to generate microwaves and put them into a rectangular waveguide. On the other side of the waveguide there is a layer of the material which is terminated by a short circuit. To calculate the dielectric constant we have to measure the standing wave pattern inside the waveguide with a movable probe. In this experiment we'll use tert-Butyl chloride, a flammable substance with a melting point of $-26°$C. At the wavelength of microwaves the dielectric constant of it shows a sudden increase when warmed up to about $-60°$C. Therefore we're going to cool the substance down to $-70°$C, let the temperature slowly increase and do the measurements.

2 Theory

The electromagnetic waves in this experiment are propagating in a rectangular metallic conductor. We are going to take a closer look at at this propagation by solving Maxwell's equations in a rectangular waveguide (assuming there is a vacuum in the waveguide, i.e. no charges or currents):

$$\mathrm{div}\vec{E} = 0 \qquad \mathrm{rot}\vec{E} = -\mu\frac{\partial\vec{H}}{\partial t}$$

$$\mathrm{div}\vec{H} = 0 \qquad \mathrm{rot}\vec{H} = \varepsilon\frac{\partial\vec{E}}{\partial t}$$

Electromagnetic Waves

In this experiment, the emitter of the microwaves produces a sinusoidal wave, therefore the field $A(x, y, z, t)$ also has a sine-shaped time dependence, which can be separated in the following way:

$$A(x, y, z, t) = A(x, y, z) \cdot \mathrm{e}^{i\omega t} \tag{1}$$

The wave can now be separated in its real and imaginary part, both have to fulfill Maxwell's equations.

Using $\vec{H}(\vec{r}, t) = \vec{H}(\vec{r})\mathrm{e}^{i\omega t}$ and Maxwell's equations we can write:

$$\begin{aligned} \vec{\nabla} \times \vec{E} &= - i\omega\mu\vec{H} \\ \vec{\nabla} \times \vec{H} &= i\omega\epsilon\vec{E} \end{aligned} \tag{2}$$

Thus, since $\vec{\nabla} \times (\vec{\nabla} \times \vec{V}) = \vec{\nabla} \cdot (\vec{\nabla} \cdot \vec{V}) - \vec{\nabla}^2 \vec{V}$ holds and the divergence of $\vec{E}$ and $\vec{H}$ vanishes, it follows:

$$\begin{aligned} \vec{\nabla}^2 \vec{E} &= -\,\omega^2 \mu \epsilon \vec{E} \\ \vec{\nabla}^2 \vec{H} &= -\,\omega^2 \mu \epsilon \vec{H} \end{aligned} \tag{3}$$

Now we take a look at differential equations for the z-components of $\vec{E}$ and $\vec{H}$:
By using an ansatz of separation $E_z(\vec{r}, t) = X(x)Y(y)Z(z)\mathrm{e}^{i\omega t}$ one gets:

$$\frac{X''}{X} + \frac{Y''}{Y} + \frac{Z''}{Z} = -\omega^2 \mu \epsilon \tag{4}$$

The three summands are independent, therefore they have to be constant individualy:

$$\frac{X''}{X} = c_1; \quad \frac{Y''}{Y} = c_2; \quad \frac{Z''}{Z} = c_3 \tag{5}$$

Waveguide

Now we are solving these differential equations in the rectangular waveguide of this experiment. We label the axis as follows:

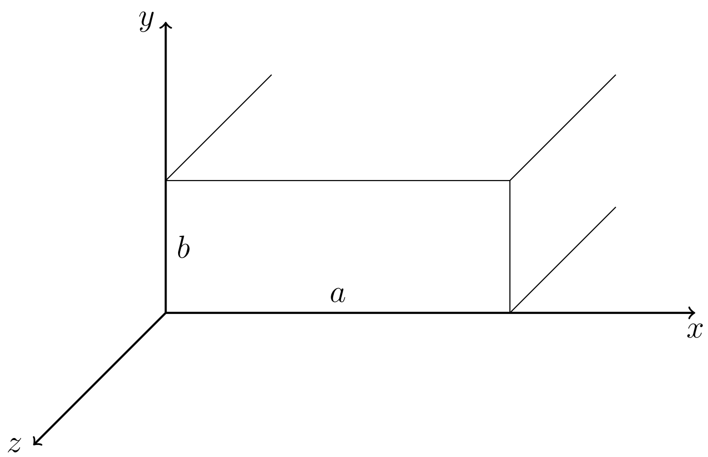

Figure 1: waveguide and choice of coordinate system

From experiments it is known that the wave propagation along the z-axis is given by

$$\vec{E} \propto \mathrm{e}^{i(\omega t - kz)}; \quad k = \frac{2\pi}{\lambda_g} \tag{6}$$

Hence

$$Z(z) = \gamma \mathrm{e}^{ikz} \tag{7}$$

In x and y direction the surface of the waveguide gives some boundary conditions for the electric field (the waveguide is assumed to be a perfect conductor, therefore the tangential component of $\vec{E}$ has to vanish on the surface of the waveguide):

$$\begin{aligned} X(0) &= X(a) = 0 \\ Y(0) &= Y(b) = 0 \end{aligned} \tag{8}$$

Thus

$$X(x) = \alpha \sin\left(\frac{m\pi}{a}x\right) \tag{9}$$

$$Y(y) = \beta \sin\left(\frac{n\pi}{b}x\right) \tag{10}$$

with $m, n \in \mathbb{N}_0$. Putting equation (9) and (10) into the ansatz one gets for the electric field (and analogous for the magnetic field):

$$E_z = E_0 \sin\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right) \mathrm{e}^{i(\omega t - kz)} \tag{11}$$

where E_0 is the product of all the constants from the previous equations. For H_z we choose the same ansatz and we will come back to this later when we calculated the boundary conditions for the magnetic field:

$$H_z = H_0 \tilde{X}(x) \tilde{Y}(y) \mathrm{e}^{i(\omega t - kz)} \tag{12}$$

Now we start with the components of equation (2):

$$\begin{aligned} \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} &= -i\omega\mu H_x & \qquad \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} &= i\omega\varepsilon E_x \\ \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} &= -i\omega\mu H_y & \qquad \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} &= i\omega\varepsilon E_y \\ \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} &= -i\omega\mu H_z & \qquad \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} &= i\omega\varepsilon E_z \end{aligned}$$

In a plane of constant z the complex field components in x and y direction should be in phase, i.e. we need $A(x, y, z, t) = A(x, y) \cdot \mathrm{e}^{i(\omega t - kz)}$. Hence

$$\begin{aligned} \frac{\partial E_z}{\partial y} + ikE_y &= -i\omega\mu H_x & \qquad \frac{\partial H_z}{\partial y} + ikH_y &= i\omega\varepsilon E_x \\ ikE_x - \frac{\partial E_z}{\partial x} &= -i\omega\mu H_y & \qquad -ikH_x - \frac{\partial H_z}{\partial x} &= i\omega\varepsilon E_y \end{aligned}$$

Solving these equations for E_x, E_y, H_x and H_y yields

$$E_x = \frac{-i}{\omega^2\mu\varepsilon - k^2}\left(k\frac{\partial E_z}{\partial x} + \omega\mu\frac{\partial H_z}{\partial y}\right) \tag{13}$$

$$E_y = \frac{-i}{\omega^2\mu\varepsilon - k^2}\left(k\frac{\partial E_z}{\partial y} - \omega\mu\frac{\partial H_z}{\partial x}\right) \tag{14}$$

$$H_x = \frac{-i}{\omega^2\mu\varepsilon - k^2}\left(-\omega\varepsilon\frac{\partial E_z}{\partial y} + k\frac{\partial H_z}{\partial x}\right) \tag{15}$$

$$H_y = \frac{-i}{\omega^2\mu\varepsilon - k^2}\left(\omega\varepsilon\frac{\partial E_z}{\partial x} + k\frac{\partial H_z}{\partial y}\right) \tag{16}$$

From these equations we can see that the transverse components of both the electric and the magnetic field can be determined from only the axial components (E_z and H_z).

TEM-waves

We can find 2 special types of waves, transverse magnetic and transverse electrical waves

(a) TE-mode: $E_z \equiv 0$

(b) TM-mode: $H_z \equiv 0$

(c) TEM-mode: $E_z \equiv 0$ and $H_z \equiv 0$

First we take a look at the TE-mode where $E_z \equiv 0$. The normal component of $\vec{E}$ has to vanish at the surface of the metal, therefore we can find new boundary conditions for $\vec{H}$ by using equation (13) and (14):

$$\frac{\partial H_z}{\partial y}(x, 0, z, t) = 0 \qquad \frac{\partial H_z}{\partial x}(0, y, z, t) = 0$$

$$\frac{\partial H_z}{\partial y}(x, b, z, t) = 0 \qquad \frac{\partial H_z}{\partial x}(a, y, z, t) = 0$$

With these boundary condition we can find H_z analogously to E_z. This allows us to write down the general form of the TE-wave:

$$E_x = \frac{iH_0\omega\mu}{\omega^2\mu\varepsilon - k^2}\frac{n\pi}{b}\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)\mathrm{e}^{i(\omega t - kz)} \tag{17}$$

$$E_y = \frac{-iH_0\omega\mu}{\omega^2\mu\varepsilon - k^2}\frac{m\pi}{a}\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\mathrm{e}^{i(\omega t - kz)} \tag{18}$$

$$E_z \equiv 0 \tag{19}$$

$$H_x = \frac{iH_0 k}{\omega^2\mu\varepsilon - k^2}\frac{m\pi}{a}\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\mathrm{e}^{i(\omega t - kz)} \tag{20}$$

$$H_y = \frac{iH_0 k}{\omega^2\mu\varepsilon - k^2}\frac{n\pi}{b}\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)\mathrm{e}^{i(\omega t - kz)} \tag{21}$$

$$H_z = H_0\cos\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\mathrm{e}^{i(\omega t - kz)} \tag{22}$$

Plugging this into equation (3) yields

$$-\left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2 - k^2 = -\omega^2\mu\varepsilon$$

$$\Rightarrow \quad k^2 = \omega^2 \mu \varepsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2 \tag{23}$$

For a wave to propagate we need k to be real, i.e. for a given a, b, μ and ε there is a cutoff frequency f_c for each mode (m,n)

$$f_c = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{\mu\varepsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} \tag{24}$$

In this experiment we are using a waveguide with $a = 2.28\,\text{cm}$ and $b = 1.02\,\text{cm}$ (with $\mu = \varepsilon = 1$):

Table 1: cutoff frequencies for modes (m,n)

m	n	f_c(GHz)
1	0	6.6
2	0	13.1
0	1	14.8
1	1	16.2

The Klystron in this experiment produces microwaves with a frequency of $f \approx 10.2\,\text{GHz}$, that means that only one mode ($m = 1$, $n = 0$) is allowed. Therefore the wave has the following form:

$$\begin{aligned} E_x &\equiv 0 & H_x &= \frac{iH_0 ka}{\pi} \sin\left(\frac{\pi}{a}x\right) \mathrm{e}^{i(\omega t - kz)} \\ E_y &= \frac{-iH_0 \omega\mu a}{\pi} \sin\left(\frac{\pi}{a}x\right) \mathrm{e}^{i(\omega t - kz)} & H_y &\equiv 0 \\ E_z &\equiv 0 & H_z &= H_0 \cos\left(\frac{\pi}{a}x\right) \mathrm{e}^{i(\omega t - kz)} \end{aligned} \tag{25}$$

Damped Wave

In this experiment the electromagentic waves have to travel through a medium with a complex dielectric constant and therefore get damped. To describe the motion of the wave through a medium we use the ansatz:

$$Z(z) = \gamma \mathrm{e}^{-ik_1 z} \mathrm{e}^{-k_2 z} = \gamma \mathrm{e}^{-iKz} \tag{26}$$

where $K = k_1 - ik_2$. Since we assume $\mu = 1$ for the materials we are using in this experiment, we can see from equation 23 that now ε has to have both a real and a complex part:

$$\varepsilon = \varepsilon_1 - i\varepsilon_2 \tag{27}$$

Plugging ε and K into equation 23 we get

$$\begin{aligned} k_1^2 - k_2^2 =& \omega^2 \mu \varepsilon_1 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2 \\ 2k_1 k_2 =& \omega^2 \mu \varepsilon_2 \end{aligned} \tag{28}$$

For simplicity, we introduce the *relative* dielectric constant ε_r

$$\varepsilon = \varepsilon_0 \cdot \varepsilon_r \qquad \varepsilon_r = \frac{\varepsilon_1}{\varepsilon_0} - i\frac{\varepsilon_2}{\varepsilon_0} \tag{29}$$

where $\varepsilon_0 = 8.85 \cdot 10^{-12}\mathrm{AsV^{-1}m^{-1}}$. Further we introduce the wavelength λ_g in the waveguide

$$\lambda_g = \frac{2\pi}{k} \tag{30}$$

which is measured in the air-filled part of the waveguide where we assume k to be real (no attenuation).

Reflection and Standing Waves

In this experiment we are using a waveguide which is terminated by a short circuit. In the second part we place a medium in front of that short circuit. The following figure shows the conditions:

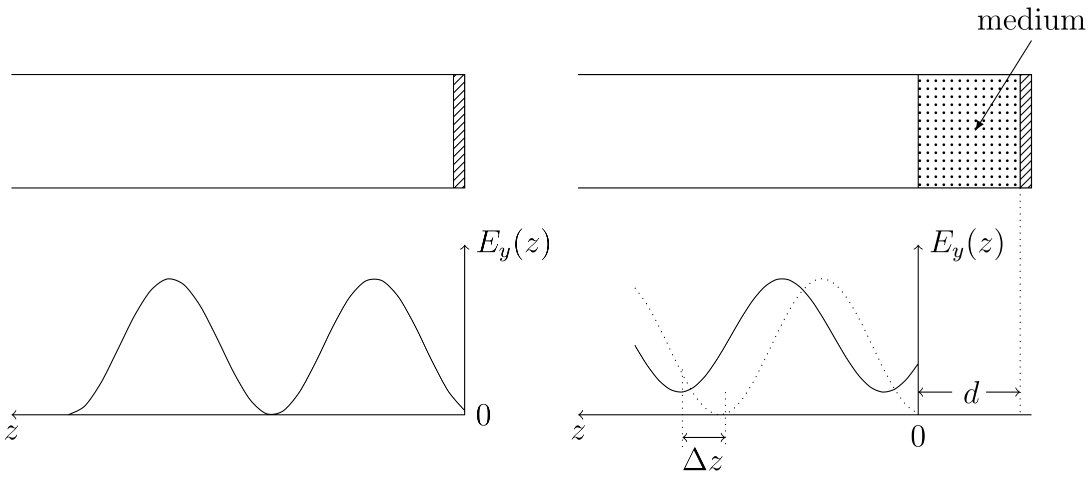

Figure 2: terminated waveguide with and without medium

We are going to call the area without medium I, the part of the waveguide with medium II. The electric and magnetic fields in this scenario can be obtained by taking the superposition of the incoming and the reflected wave.

In area I:

$$\begin{aligned} E_y =& -iH_0\omega\mu_I \frac{a}{\pi} \sin\left(\frac{\pi}{a}x\right)\left(\mathrm{e}^{-ik_I z} + R\mathrm{e}^{ik_I z}\right)\mathrm{e}^{i\omega t} \\ H_x =& \quad iH_0 k_I \frac{a}{\pi} \sin\left(\frac{\pi}{a}x\right)\left(\mathrm{e}^{-ik_I z} - R\mathrm{e}^{ik_I z}\right)\mathrm{e}^{i\omega t} \end{aligned} \tag{31}$$

where $R = |R|\mathrm{e}^{i\delta}$ and the index I or II specifices the area.

In area II:

$$\begin{aligned} E_y &= -\, iH_0'\omega\mu_{II}\frac{a}{\pi}\sin\left(\frac{\pi}{a}x\right)\left(\mathrm{e}^{-ik_{II}z} - \mathrm{e}^{-ik_{II}d}\mathrm{e}^{ik_{II}(z-d)}\right)\mathrm{e}^{i\omega t} \\ H_x &= \quad iH_0'k_{II}\frac{a}{\pi}\sin\left(\frac{\pi}{a}x\right)\left(\mathrm{e}^{-ik_{II}z} + \mathrm{e}^{-ik_{II}d}\mathrm{e}^{ik_{II}(z-d)}\right)\mathrm{e}^{i\omega t} \end{aligned} \tag{32}$$

At the interface we have the following continuity conditions for the transverse and normal components of the electromagnetic field:

$$\begin{aligned} E_{I,t} &= E_{II,t} \\ H_{I,t} &= H_{II,t} \end{aligned} \tag{33}$$

Since x and y are both tangential to the direction of propagation, for $z = 0$ we can find the following relations:

$$\begin{aligned} \mu_I H_0(1+R) &= \mu_{II}H_0'\left(1 - \mathrm{e}^{-i2k_{II}d}\right) \\ k_I H_0(1-R) &= k_{II}H_0'\left(1 + \mathrm{e}^{-ik_{II}d}\right) \end{aligned} \tag{34}$$

For air and non-magnetic materials we can approximate $\mu_I \approx 1 \approx \mu_{II}$ and since the dampening of the wave in air is neglectable we have $k_I \in \mathbb{R}$. Hence:

$$\frac{-i}{k_I d}\,\frac{1+R}{1-R} = \frac{\tanh(ik_{II}d)}{ik_{II}d} \tag{35}$$

In the analysis of the experiment we will use this trancendent equation to find k_{II}.

VSWR and Δz

The diode in the slotted line of this experiment measures a quadratic signal of the standing wave, i.e. $E_y(z)^2$. For a terminated circuit without a dielectric, we have:

$$\begin{aligned} E_y(z) &= E_0\mathrm{e}^{i\omega t}\left(\mathrm{e}^{-ikz} + \mathrm{e}^{ikz}\right) \\ \Rightarrow E_y^2(z) &= 4E_0^2\sin^2(kz) \end{aligned} \tag{36}$$

Now with a dielectric medium in the waveguide, the reflected wave is attenuated (no total destructive interference) and therefore the value at the minima of the standing wave is not zero! Also note that the minima get displaced by Δz (see fig. 2) against the positions without a dielectric. With the following notation for the complex reflection coefficient $R = |R|\mathrm{e}^{i\delta}$ we have:

$$\begin{aligned} E_y(z) &= E_0\mathrm{e}^{i\omega t}\left(\mathrm{e}^{-ikz} + |R|\mathrm{e}^{i\delta+ikz}\right) \\ \Rightarrow E_y^2(z) &= E_0^2\left(1 + |R|^2 + 2|R|\cos(2kz+\delta)\right) \end{aligned} \tag{37}$$

Thus

$$\begin{aligned} E_{min} &= (1-|R|)E_0 \\ E_{max} &= (1+|R|)E_0 \end{aligned} \tag{38}$$

We now introduce the voltage standing wave ratio (VSWR) which can be directly measured in this experiment:

$$\text{VSWR} = \frac{E_{max}}{E_{min}} = \frac{1+|R|}{1-|R|} \tag{39}$$

The second observable which has to be measured is Δz, the dispalcement of the minima (see fig 2). By taking into account the phase shift of the reflected wave at the metal, we can find:

$$\delta = \pi - 2k\Delta z \tag{40}$$

$$R = \frac{\text{VSWR}-1}{\text{VSWR}+1}\mathrm{e}^{i(\pi-2k\Delta z)} = \frac{\text{VSWR}-1}{\text{VSWR}+1}\mathrm{e}^{i\delta} \tag{41}$$

For further reading see [2] and [3].

3 Experimental Setup

3.1 Overview

In this experiment we are going to determine the complex dielectric constant of tert-Butyl chloride at different temparatures by measuring a standing wave pattern of microwaves.

The Klystron on the left of the experimental setup (see fig. 3) produces microwaves (with $f \approx 10.2\,\text{GHz}$) with an electron beam. This generates a lot of heat, therefore the water cooling should be turned on when the Klystron is running. The wave attenuator then decreases the intensity of the generated waves.

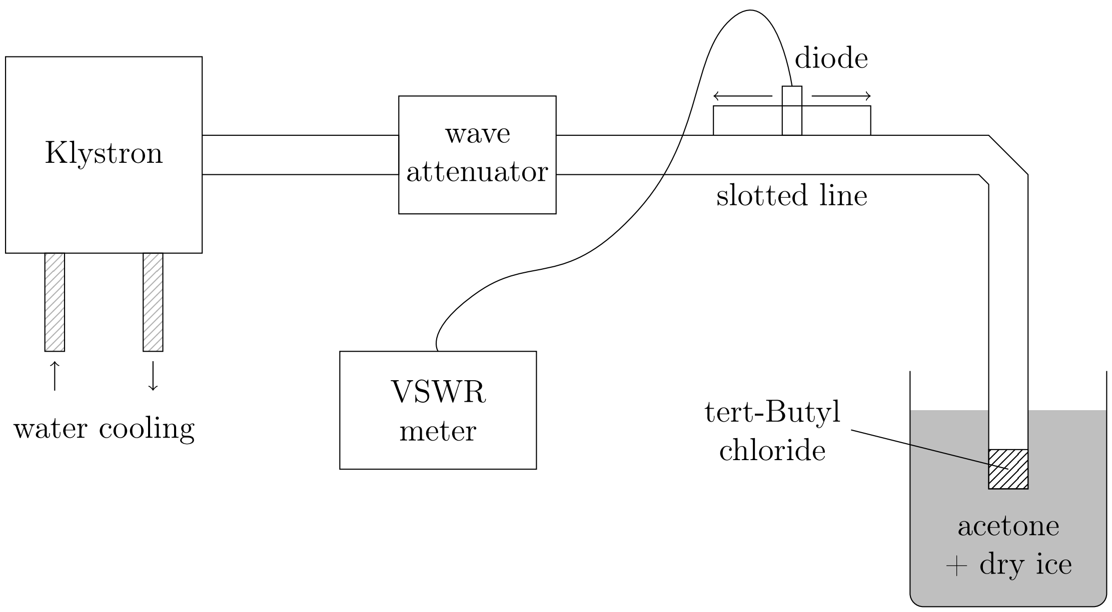

Figure 3: experimental setup

The microwaves travel to the end of the rectangular waveguide, get reflected and form a standing wave. With the movable diode in the slotted line the voltage standing wave ration (VSWR) at different positions can be observed, i.e. the minima can be located.

For the first measurement, no t-Butyl chloride and acetone dry ice mixture is needed. In this measurement, the properties of the waveguide itself are determined.

For the second measurement, the removable part of the waveguide is partially filled with tert-Butyl chloride. The more of this chemical is used, the bigger the displacement of the minima of the standing wave is. If too much is used, the displaced minima cannot be assigned to the previous minima anymore. A good value for the height of t-Butyl chloride is $\approx 0.5\,\text{cm}$.

3.2 How to measure VSWR and Δz

The easiest way to measure Δz is to first measure the absolute position of the minima in the vacuum. To find a minimum, the dial can be used to move the diode along the slotted line until the VSWR-meter shows a minimal amplitude, then the exact position can be read off the vernier scale. Keep in mind that the position will be shifted by introducing the dielectric, so do not choose minima too close to the egde of the measuring scale. Now for every measurement with the medium the absolute position of the minima can be measured and the differences ($= \Delta z$) can be calculated afterwards.

With a VSWR-meter it is hard to measure high VSWR values directly, therefore we are going to use a sligthly different method. In the experiment without dielectrica, the minimum of the standing wave pattern of the electric field is 0, therefore $\text{VSWR} = E_{max}/E_{min}$ is going to infinity. If we insert a dielectrica in the waveguide, the value of E_{min} becomes finite and so does VSWR. We can now adjust the VSWR-meter by using the buttons with steps of 10dB on the right side and rotating the GAIN-button so that the minimum is exactly at 3.0dB. Now we can use the dial to move the probe to find the distance ν between the two positions (left and right of the minimum) where we have 0dB. Again, this can be done very precise by using the Vernier scale. The VSWR-value of the minimum can then be calculated by:

$$\text{VSWR} = \frac{\lambda_g}{\pi\nu} \tag{42}$$

4 Preparing the Measurement

First, the assistent should organize the dry ice from department of chemistry. It is stored in a huge box from the company PanGas and can be transported easily in a styrofoam box. Then follow this guide step-by-step:

- turn on the timer of the water cooling

- turn on the Klystron, adjust the settings as shown in fig. 4
- turn on the SWR-meter, for settings see 5
- wait for a few minutes until the ampere-meters on the Klystrons show stable values
- determine $\frac{\lambda_g}{2}$ by measuring the distance between two adjacent minima, use the dial to move the probe and the vernier scale to read off the values
- calculate $k = \frac{2\pi}{\lambda_g}$
- measure the absolute position of the two minima you want to observe
- use an Eppendorf pipette to fill an amount V of t-Butyl chloride into the removable part of the waveguide, calculate thickness $d = \frac{V}{a \cdot b}$ of the chemical, where a and b are the dimensions of the waveguide. choose V such that $d = 0.5 \pm 0.1\,\mathrm{cm}$
- put the removable part of the waveguide back in place and fix it with the screws
- prepare the acetone and dry ice mixture in a dewar vessel (don't use too much dry ice, it takes very long to heat up then)
- use a lab-jack to lift the vessel from below the part of the waveguide that is filled with tert-Butyl chloride such that the waveguide immerses a few cm into the cooling mixture
- place the sensor of the thermometer in the cooling mixture

5 Measurement

In this experiment the complex dielectric constant should be measured between $-70°\mathrm{C}$ and $-50°\mathrm{C}$ in small steps. Therefore we start with the cooling mixture which is slightly cooler than $-70°\mathrm{C}$ and wait for it to heat up. The temperature can be read off the thermometer. Now the acutal measurement has to be performed:

- note down the current temperature
- move the probe to the first minimum you want to observe, measure ν and Δz as described in section 3.2
- move the probe to the second minimum, measure ν and Δz again

This procedure has to be repeated every 0.5 or 1 degree that the cooling mixture has heated up. If the heating rate is too high, one gets very poor measurement results. Stop with the measurement once you reached $-50°\mathrm{C}$.

Now the experiment can be shut down, the tert-Butyl chloride can be washed down the sink with some water. Wait for the cooling mixture (i.e. the acetone) to heat up to room temperature, then put it carefully back in the bottle for the used acetone.

6 Analysis

For every measurement point the following 10 steps have to be applied:

1. calculate VSWR $= \frac{\lambda_g}{\pi\nu}$
2. calculate $|R| = \frac{\text{VSWR}-1}{\text{VSWR}+1}$
3. calculate $\delta = \pi - 2\,k\,\Delta z$
4. calculate $q = \frac{-i}{k\,d} \cdot \frac{1+|R|\mathrm{e}^{i\delta}}{1-|R|\mathrm{e}^{i\delta}}$
5. solve the trancendental equation $\frac{\tanh(z)}{z} = q$ numerically. This can be done with Mathematica: FindRoot[Tanh[z]/z == q, {z, x + i * y}], with starting values between $0.1{+}5{*}i$ for low temperatures and $0.5{+}11{*}i$ for the higer temperatures. Alternatively the equation can be solved with MatLab by initializing a matrix with e.g. all values between $0 + 0 * i$ and $0.2 + 10 * i$ and find the best fitting matrix element.
6. calculate $k_2 = \frac{z}{i\,d}$
7. calculate $\varepsilon_1 = \frac{\mathrm{Re}(k_2)^2 - \mathrm{Im}(k_2)^2 + \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}{\varepsilon_0\mu_0\omega^2}$ where a and b are the dimensions of the waveguide and m and n are the modes of the TE-waves (in this case $m = 1$, $n = 0$ if you choose a to be the greater dimension) and $\omega = 2\pi \cdot f$
8. caluclate $\varepsilon_2 = \frac{2 * \mathrm{Re}(k_2)\mathrm{Im}(k_2)}{\varepsilon_0\mu_0\omega^2}$
9. calculate the complex dielectric constant $\varepsilon = \varepsilon_1 - i \cdot \varepsilon_2$

7 Warnings

In this experiment we are going to use acetone and tert-Butyl chloride. Use latex gloves when working with the chemicals and do not bring it in contact with your eyes. Also the vapour shouldn't be directly inhaled and if there is too much aceton vapour in the air make sure that the windows are wide open. Aceton can solute plastic materials, e.g. styrofoam (Styropor) or PET.

Be careful with the gaseous CO_2 that sublimates out of the dry ice. Make sure that the room is well vantilated.

Be especially careful with the cooling mixture: When getting in contact with skin it can cause cold burns. Also, the mixture of dry ice and aceton can start to produce bubbles and splash around if one adds too much dry ice at once.

8 Exercises

1. Look at these questions in advance:
   - How does a Klystron work?
   - What is the meaning of the real and the complex part of the dielectric constant?
2. Give a short overview of the theory
3. Measure the dielectric constant at two minima from −70°C to −50°C in small steps
4. Plot for both minima the real part and the imaginary part of the dielectric constant in a diagram
5. Discuss your results
6. At some point there is a sudden change of the dielectric constant. Can you explain this phenomenon? (HINT: The molecule has a dipol moment and the degrees of freedom of rotation can change with temperature, further reading [4])

References

[1] Frank Peis, *Komplexe DK von t-Butylchlorid*, 1988

[2] J. D. Jackson, *Classical Electrodynamics*

[3] A. R. von Hippel, *Dielectric Materials and Applications*, 1954, p.13 et seq.

[4] Powles, Williams and Smith, *Dielectric Dispersion in the Microwave Region of Six Tetrasubstituted Methanes in the Solid State*, 1952

Appendix

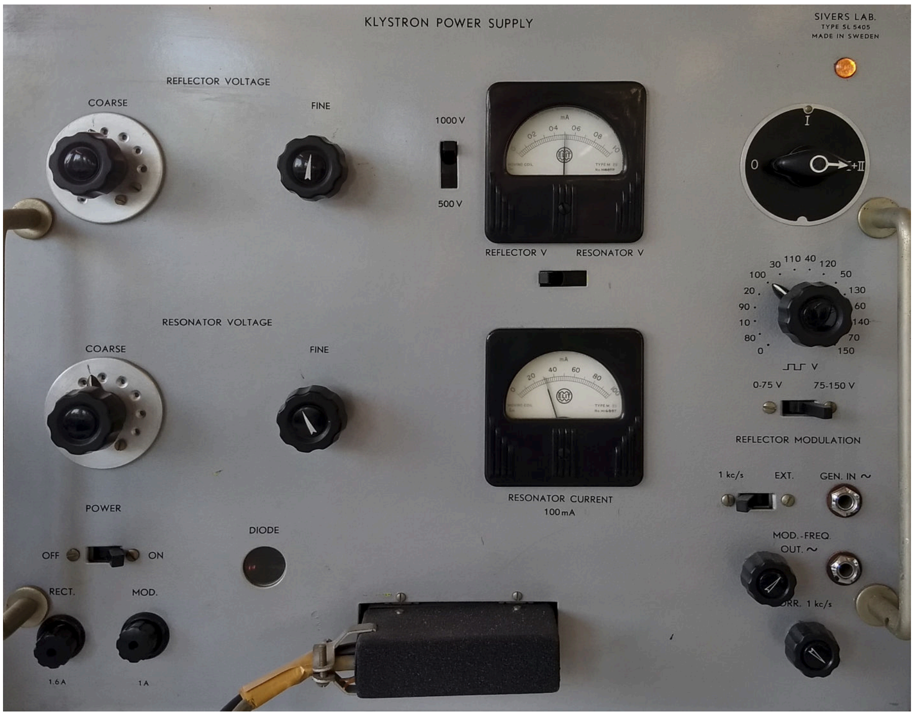

Figure 4: Settings of the Klystron

Figure 5: Settings of the SWR-meter